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NEW AND OLD PROOFS OF THE PYTHAGOREAN THEOREM.

By **BENJ. F. YANNEY, A. M.**, Mount Union College, Alliance, Ohio, and **JAMES A. CALDERHEAD, B. Sc.**, Curry University, Pittsburg, Pennsylvania.

[Continued from April Number.]

VII. Let ABC be a $\triangle$ right angled at C. Produce BC making $BD=BA$. Join DA. From E, the middle point of CD, draw a perpendicular meeting DA, as at F. Join FB. $\triangle ADC$ is similar to $\triangle BFE$.

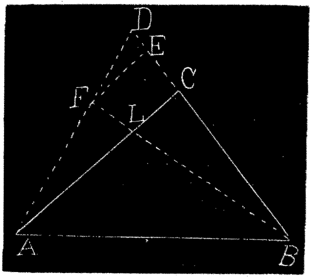

Fig. 7.

$\therefore AC : BE :: DC : FE$.

$\therefore AC : BC+(AB-BC)\div 2 :: AB-BC : AC\div 2$.

$\therefore \overline{AB}^2=\overline{AC}^2+\overline{BC}^2$.

Note.—This proof is credited to Hoffman.

VIII. Triangles BDF, BFE, and FDE, are similar.

Letting $BD=BA=c$, $AC=b$, $BC=a$, $BE=\frac{a+c}{2}$, $DE=\frac{c-a}{2}$, $FE=\frac{b}{2}$, $DF=x$, $BF=y$, we obtain the following:

(1). $\frac{c-a}{2} : x :: x : c. \quad \therefore x^2=\frac{c(c-a)}{2}$ ....1.

(2). $\frac{c-a}{2} : x :: \frac{b}{2} : y. \quad \therefore bx=(c-a)y$ ....2.

(3). $x : c :: \frac{b}{2} : y. \quad \therefore xy=\frac{bc}{2}$ ....3.

(4). $\frac{c-a}{2} : \frac{b}{2} :: \frac{b}{2} : \frac{c+a}{2}. \quad \therefore c^2-a^2=b^2$ ....4.

(5). $\frac{c-a}{2} : \frac{b}{2} :: x : y. \quad \therefore bx=(c-a)y$ ....2.

(6). $\frac{b}{2} : \frac{c+a}{2} :: x : y. \quad \therefore by=(c+a)x$ ....5.

(7). $\frac{c+a}{2} : y :: y : c. \quad \therefore y^2=\frac{c(c+a)}{2}$ ....6.

(8). $\frac{c+a}{2} : y :: \frac{b}{2} : x. \quad \therefore by=(c+a)x$ ....5.

(9). $y : c :: \frac{b}{2} : x. \quad \therefore xy=\frac{bc}{2}$ ....3.

From 4 we get $c^2=a^2+b^2$.

The set 2 and 5 gives the same result. But equation 2 may come from proportion (2) or (5), and 5 from (6) or (8), thus making four proofs for this set.

The following sets of three equations furnish fourteen proofs, since each set can come from two or more sets of three proportions: 1, 2, 6; 1, 3, 5; 1, 3, 6; 1, 5, 6; 2, 3, 6. Total number of proofs for this method is 19.

IX. Comparing the triangles BDF, BFE, ADC, BLC, and ALF, Fig. 7, we may put the result in the following condensed form:

$$DF=x : EF=\tfrac{1}{2}b : DC=c-a : LC=z : FL=y-v$$

$$:: FB=y : EB=\tfrac{1}{2}(c+a) : AC=b : CB=a : AF=x$$

$$:: BD=c : FB=y : AD=2x\ LB=v : AL=b-z.$$

From this we easily may derive thirty different simple proportions, which give twenty-seven different equations. Some idea of the number of proofs that may be obtained from different sets of these equations, can be formed from the fact that there are 17550 sets of four equations, to say nothing of sets of three and of five. Of course, many of the sets must be rejected for reasons stated fully in V. We leave details to the reader.

X. Suppose the theorem true. Then $\overline{AB}^2=\overline{AC}^2+\overline{BC}^2$, $\overline{BC}^2=\overline{CD}^2+\overline{BD}^2$, and $\overline{AC}^2=\overline{AD}^2=\overline{CD}^2$.

$\therefore\ \overline{AB}^2=\overline{AD}^2+2\overline{CD}^2+\overline{BD}^2$.

But $\overline{CD}^2=AD\cdot BD$.

$\therefore\ \overline{AB}^2=\overline{AD}^2+2AD\cdot BD+\overline{BD}^2$.

$\therefore\ AB=AD+BD$, which is true.

$\therefore$ The supposition is true.

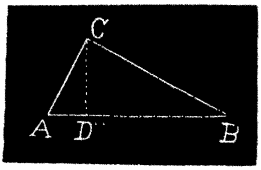

Fig. 1.

Note.—This method is credited to Hoffman.

XI. In Fig. 1, $\overline{AB}^2<, =,$ or $>\ \overline{AC}^2+\overline{BC}^2$. Suppose it less. Then, since $\overline{AB}^2=(AD+DB)^2=(\overline{CD}^2\div DB+DB)^2$, and $\overline{AC}^2=(\overline{CD\cdot BC}\div DB)^2$,

$$(\overline{CD}^2\div DB+DB)^2<(\overline{CD\cdot BC}\div DB)^2+\overline{BC}^2.$$

$\therefore\ (\overline{CD}^2+\overline{DB}^2)^2<\overline{BC}^2(\overline{CD}^2+\overline{DB}^2)$.

$\therefore\ \overline{BC}^2>\overline{CD}^2+\overline{DB}^2$, which is absurd. For were the supposition true, we should have $\overline{BC}^2<\overline{CD}^2+\overline{DB}^2$, as can easily be shown.

Similarly the supposition that $\overline{AB}^2>\overline{AC}^2+\overline{BC}^2$ can be proven false.

$\therefore\ \overline{AB}^2-\overline{AC}^2+\overline{BC}^2$.

XII. $BC=a : EF=x : DF=y : DE=z$

$:: AC=b : AF=v : EF=x : AE=w$

$:: AB=c : AE=w : ED=z : AD=v+y.$

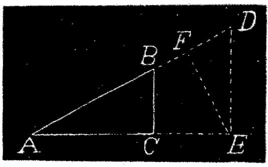

Fig. 8.

The above condensed form is self-explanatory, as are also the two following.

We leave the selection of simple proportions, the derivation and solution of consequent equations, as an exercise for the interested reader.

XIII. $BC=a : DE=x : DL=y,$

$:: AB=b : AE=z : LF=FE=v,$

$:: AB=c : AD=v+y : DF=x-v.$

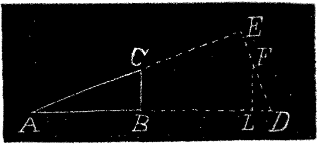

Fig. 9.

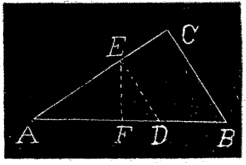

Fig. 10.

XIV. $BC=a : ED=EC=x : FD=y : EF=z,$

$:: AC=b : AE=b-x : EF=z : AF=v,$

$:: AB=c : AD=v+y : ED=x : AE=b-x.$

[To be Continued.]

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INTRODUCTION TO SUBSTITUTION GROUPS.

By G. A. MILLER, Ph. D., Leipzig, Germany.

[Continued from May Number.]

Each of the regular groups of degree six contains only one subgroup of the type (*abc.def*). Since no substitution of the form *abcde* can transform this into itself the group of order 30 is impossible.

If a group of order 60 exists it must contain six subgroups of the type $(abcde)_{10}$. We may assume that it contains $(abcde)_{10}\equiv G_f$. G_b must then contain *ac.de* and a substitution of the type *abcde* which contains the letters *a*, *c*, *d*, *e*, *f*. We may assume that this substitution is $acd_1e_1f_1$. It is then necessary that $ac.de.acd_1e_1f_1=af_1e_1d_1c.ac.de$. Hence

$$acd_1e_1f_1=acdfe \text{ or } acefd.$$